The effects of nonextensive statistics on fluctuations investigated in event-by-event analysis of data

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Abstract

We investigate the effect of nonextensive statistics as applied to the chemical fluctuations in high-energy nuclear collisions discussed recently using the event-by-event analysis of data. It turns out that very minuite nonextensitivity changes drastically the expected experimental output for the fluctuation measure. This results is in agreement with similar studies of nonextensity performed recently for the transverse momentum fluctuations in the same reactions.

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Some time ago a novel method of investigation of fluctuations in even-by-event analysis of high energy multiparticle production data has been proposed [1]. It consists in defining a suitable measure Φ of a given observable x being exactly the same for nucleon-nucleon and nucleus-nucleus collisions if the later are simple superpositions of the former.

$$\Phi_x = \sqrt{\frac{\langle Z^2\rangle}{\langle N\rangle}} - \sqrt{\bar{z^2}} \qquad \text{where} \qquad Z = \sum_{i=1}^{N} z_i. \tag{1}$$

Here $z_i = x_i - \bar{x}$ where $\bar{x}$ denotes the mean value of the observable x calculated for all particles from all events (the so called inclusive mean) and N is the number of particles analysed in the event. In (1) $\langle N\rangle$ and $\langle Z^2\rangle$ are averages of event-by-event observables over all events whereas the last term is the square root of the second moment of the inclusive z distribution. By construction [1] if particles are produced independently $\Phi_x = 0$.

When applied to the recent NA49 data from central $Pb-Pb$ collisions at 158 A·GeV [2] this method revealed that fluctuations of transverse momentum ($x = p_T$) decreased significantly in respect to elementary NN collisions. This has been interpreted as a possible sign of equilibration taking place in heavy ion collisions providing thus enviroment for the possible creation of quark-gluon plasma (QGP). It was quickly realised that existing models of multiparticle production are leading in that matter to conflicting statements [3]. On the other hand the use of fluctuations as a very sensitive tool for the analysis of dynamics of multiparticle reactions has been advocated since already some time [4], especially their role in searching for some special features of the QGP equation of state has been shown to be of special interest [5].

However, it was demonstrated recently in [6] that the corresponding fluctuation measure calculated for a pion gas in global equilibrium (defined within the standard extensive thermodynamic) is almost an order of magnitude greater then the experimental value. Although the recent NA49 paper [7] presents a new value, which is more like the prediction in [6], the controversy aroused around Φ resulted in a number of presentations trying to clarify and extend the meaning of the Φ variable (cf., for example, [8] and references therein[1])[2]. In the mean time the use of this

[1]For some other recent discussions of event-by-event fluctuations see [9].

[2]We would like to point here only that, if there are some additional fluctuations (not arising from quantum statistics, like those caused by the experimental errors) which add in the same way to both terms in definition (1) of Φ it would perhaps be better to use $\Phi \to \Phi^\star = \frac{\langle Z^2\rangle}{\langle N\rangle} - \bar{z^2}$ where they would cancel.

variable has been extended, so far only theoretically, to the possible study (actually planned already by NA49) of the event-by-event fluctuations of the "chemical" (particle type) composition of the final stage of high energy collisions [10, 11].

Moving apart from the above discussion we would like to follow another path of research. Namely, it was suggested recently in [12] that the extreme conditions of density and temperature occuring in ultrarelativistic heavy ion collisions can lead to memory effects and long-range colour interactions and to the presence of non-Markovian processes in the corresponding kinetic equations (cf., for example [13]). It turns out that such effects in many other branches of physics are best described phenomenologically in terms of a single parameter q by using the so called nonextensive statistics [14]. This statistics is based on new definition of q-entropy (which for $q \to 1$ coincides with the usual Boltzmann-Gibbs definition):

$$S_q = \frac{1}{q-1}\sum_{k=1}^{W} p_k \left(1 - p_k^{q-1}\right) \quad \stackrel{q\to 1}{\Rightarrow} \quad S = -\sum_{k=1}^{W} p_k \ln p_k \tag{2}$$

(defined for the probability distribution $\{p_k\}$ for a system of W microstates). Such entropy is nonextensive, i.e., for a system $(A+B)$ composed with two independent systems A and B:

$$S_q(A+B) = S_q(A) + S_q(B) + (1-q)\, S_q(A)\cdot S_q(B). \tag{3}$$

The value of $|q-1|$ characterises deviation from the extensitivity, see [12, 14] for more details. As was shown in [12], nonextensive approach with q as minuite as $q = 1.01 \div 1.015$ eliminates the abovementioned discrepancy between the first NA49 data [2] and ideal quantum gas (Boltzmann statistics) estimation of [6].

Because NA49 Collaboration plans to study also the chemical fluctuations, there have already apeared predictions concerning the expected form of the fluctuation measure Φ in this case [11]. They are based on the use of normal (i.e., Boltzmann) statistics. In the present note we have generalized it to the case of nonextensive statistics in a manner identical to that presented in [12] for the case of transverse momenta. As in [11] we have computed the Φ measure for the system of particles of two sorts, π^- and K^-, i.e., nonstrange and strange hadrons with multiplicities $\langle n_\pi\rangle$ and $\langle n_K\rangle$, respectively. Since

$$\langle N\rangle = \langle n_\pi\rangle + \langle n_K\rangle \tag{4}$$

one immediately finds that in definition (1)

$$\langle z^2\rangle = \frac{\langle n_\pi\rangle\,\langle n_K\rangle}{\langle N\rangle} \tag{5}$$

and

$$\langle Z^2 \rangle \;=\; \frac{\langle n_\pi \rangle^2 \langle n_K^2 \rangle \;+\; \langle n_\pi^2 \rangle \langle n_K \rangle^2 - \, 2\,\langle n_\pi \rangle \langle n_K \rangle}{\langle N \rangle^2}. \tag{6}$$

He have now consistently replaced the mean occupation numbers by their q-equivalents, which under some approximations, valid for small values of nonextensitivity $|1-q|$, can be expressed in the following analytical form [15]:

$$\langle n \rangle_q \;=\; \left\{ [1 \,+\, (q-1)\beta(E-\mu)]^{1/(q-1)} \pm 1 \right\}^{-1}, \tag{7}$$

where $\beta = 1/kT$, μ is chemical potential and the $+/-$ sign applies to fermions/bosons. Notice that in the limit $q \to 1$ (extensive statistics) one recovers the conventional Fermi-Dirac and Bose-Einstein distributions. We shall not dwell on the details of this procedure, they are essentially the same as those discussed in [12]. It is only necessary to mention that in this approximation one retains the basic factorised formula for correlations used in [11], namely that

$$\langle n_i n_j \rangle \;=\; \langle n_i \rangle \langle n_j \rangle. \tag{8}$$

As in [12] (where Φ for transverse momenta p_T has been considered), a rather large sensitivity of predictions presented in [11] to the parameter q has been observed. According to the nonextensive statistics philosophy this fact indicates a large sensitivity to the (initial and boundary) conditions present in the ultrarelativistic heavy ion collisions and existence of some kind of memory effects in such systems, as mentioned in references [13]. Our results are presented in Figs. 1 and 2 where modifications caused by the nonextensitivity $q = 1.015$ (chosen in such a way as to fit the p_T spectra in Fig. 3, see discussion below) to the results of [11] for directly produced particles are shown. For simplicity we have restricted ourselves here only to comparison with results of [11] without resonances [3].

Notice (cf. also [12]) that, as is clearly seen in Fig. 3, the same pattern of fluctuations is already present in the transverse momentum spectra of produced secondaries, i.e., the same value of q brings new "q-thermal" curve in agreement with experiment in the whole range of p_T presented. If it would emerge also in the future data on the fluctuations of chemical composition discussed here, i.e., if parameter q would turn out to be similar (modulo experimental errors), it would signal that

[3] One can argue that resonance production belongs in our philosophy already to the nonextensive case being therefore responsible for (at least a part of) the effect leading to a nonzero $|1-q|$. This is best seen inspecting results of [11] with resonances included, which show that Φ in this case also changes sign. The use of parameter q is, however, more general as it includes all other possible effects as well.

both observables, fluctuations of which is investigated, are similarly affected by the external conditions mentioned before and that they can be easily parametrized phenomenologically by a single parameter q, i.e., by the measure of the nonextensitivity of the nuclear collision process[4].

[4]It is worth to mention here that methods of nonextensive statistics have been already used in the field of high energy physics in order to analyse some aspects of cosmic ray data [17] and to the description of hadronization in e^+e^- annihilation processes [18]. Also the recently proposed use of quantum groups in studying Bose-Einstein correlations observed in all multiparticle reactions [19] belong to that cathegory because, as was shown in [20], there is close correspondence between the deformation parameter of quantum groups and the nonextensitivity parameter of Tsallis statistics. In fact, as can be seen from [14], also works on intermittency using the so called Lévy stable distributions (for example [21]) belong to this cathegory as well.

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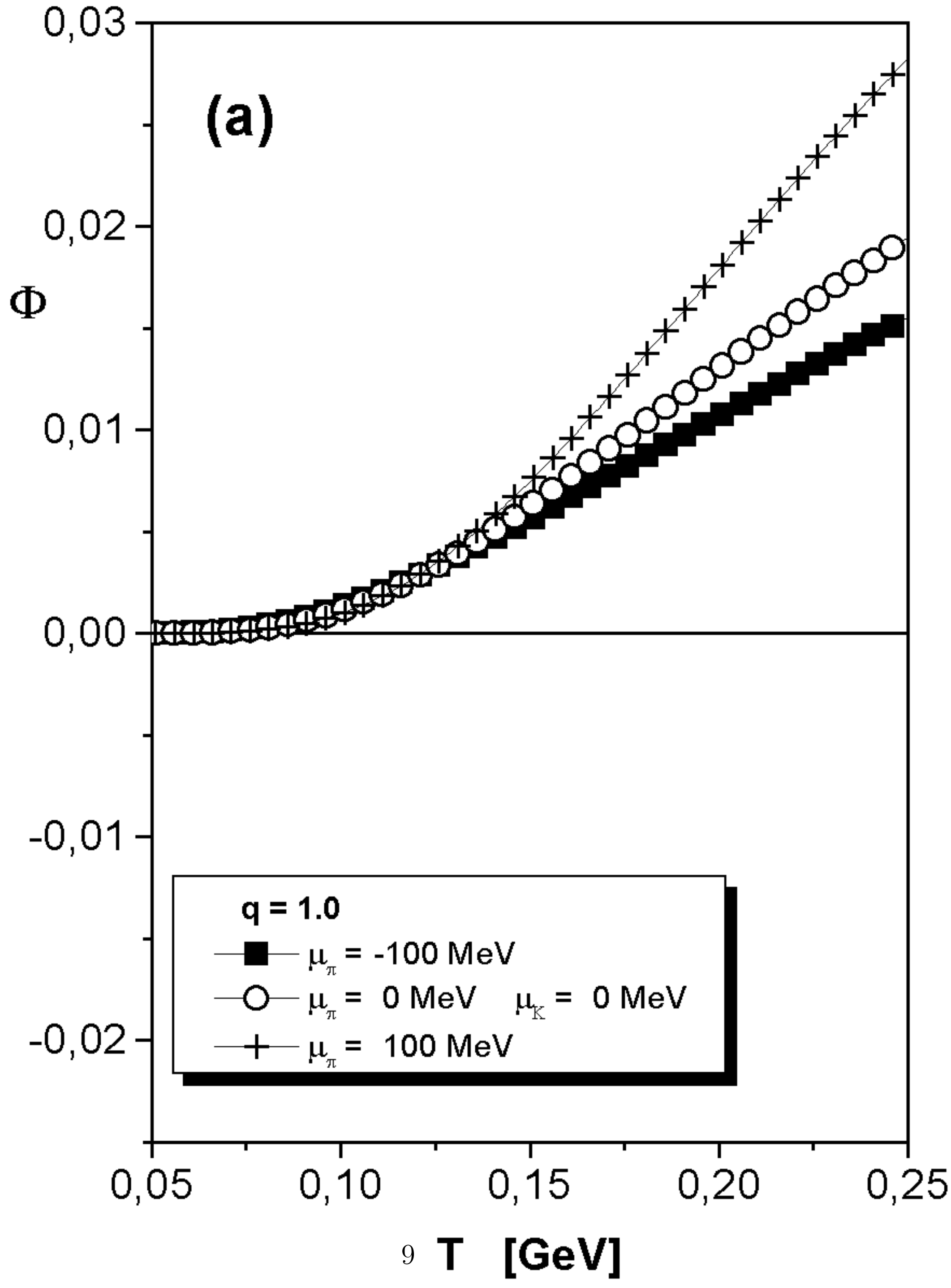
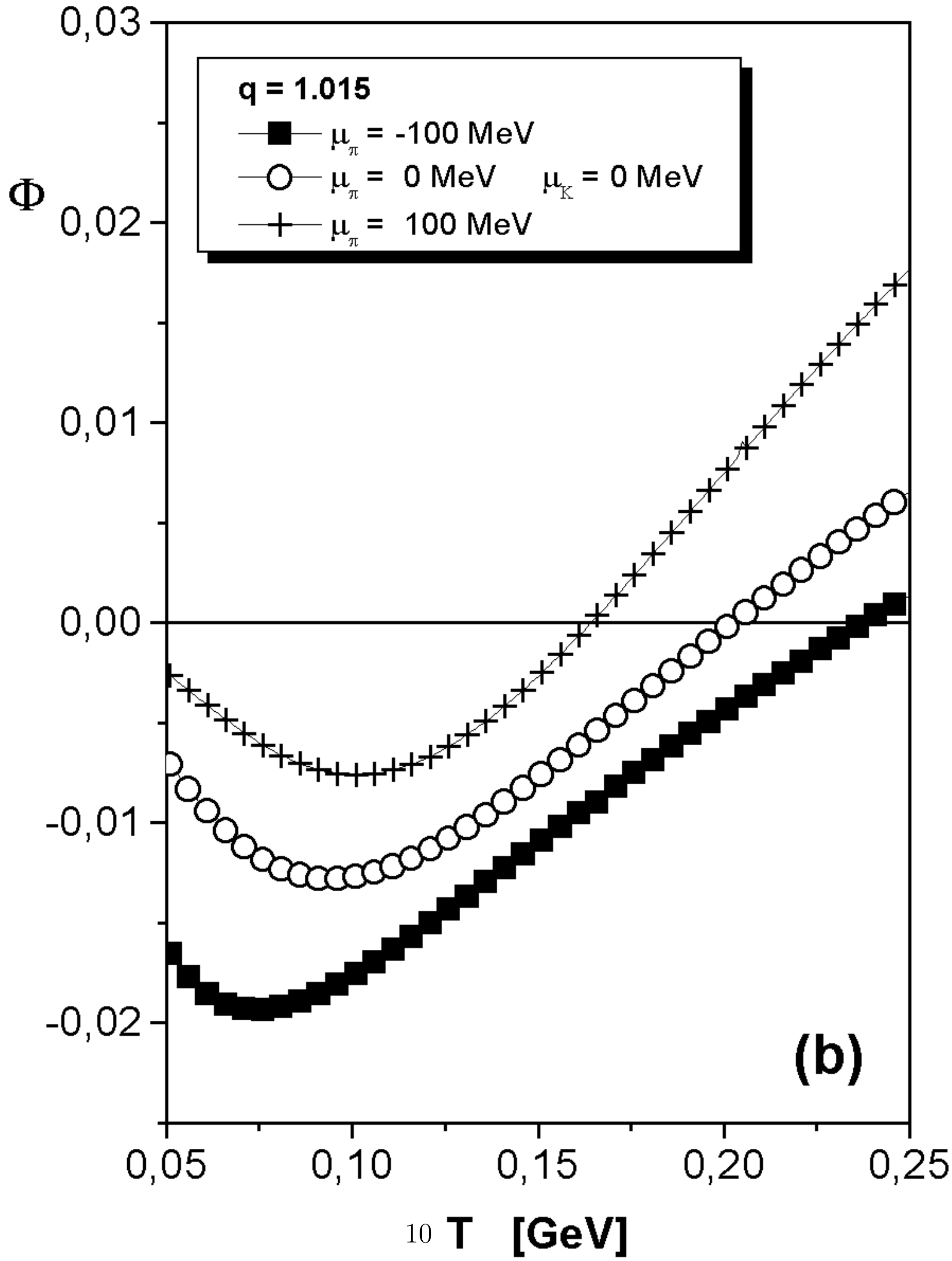
Figure Captions:

Fig. 1 Φ - measure of the kaon multiplicity fluctuations (in the $\pi^- K^-$ system of particles) as a function of temperature for three values of the pion chemical potential. The kaon chemical potential vanishes. The resonances are neglected. (a) - results of [11] (in linear scale); (b) - our results for $q = 1.015$.

Fig. 2 Φ - measure of the kaon multiplicity fluctuations (in the $\pi^- K^-$ system of particles) as a function of temperature for three values of the kaon chemical potential. The pion chemical potential vanishes. The resonances are neglected. (a) - results of [11] (in linear scale); (b) - our results for $q = 1.015$.

Fig. 3 The results for p_T distribution: notice that $q = 1.015$ results describes also the tail of distribution not fitted by the conventional exponent (i.e., $q = 1$ in our case, cf. also [12]). Data are taken from [16].


(a)
Φ
0,03
0,02
0,01
0,00
-0,01
-0,02
q = 1.0
μπ = -100 MeV
μπ = 0 MeV μK = 0 MeV
μπ = 100 MeV
0,05
0,10
0,15
0,20
0,25
T [GeV]



0,03
0,02
0,01
0,00
-0,01
-0,02
Φ
q = 1.015
μπ = -100 MeV
μπ = 0 MeV
μK = 0 MeV
μπ = 100 MeV
(b)
0,05
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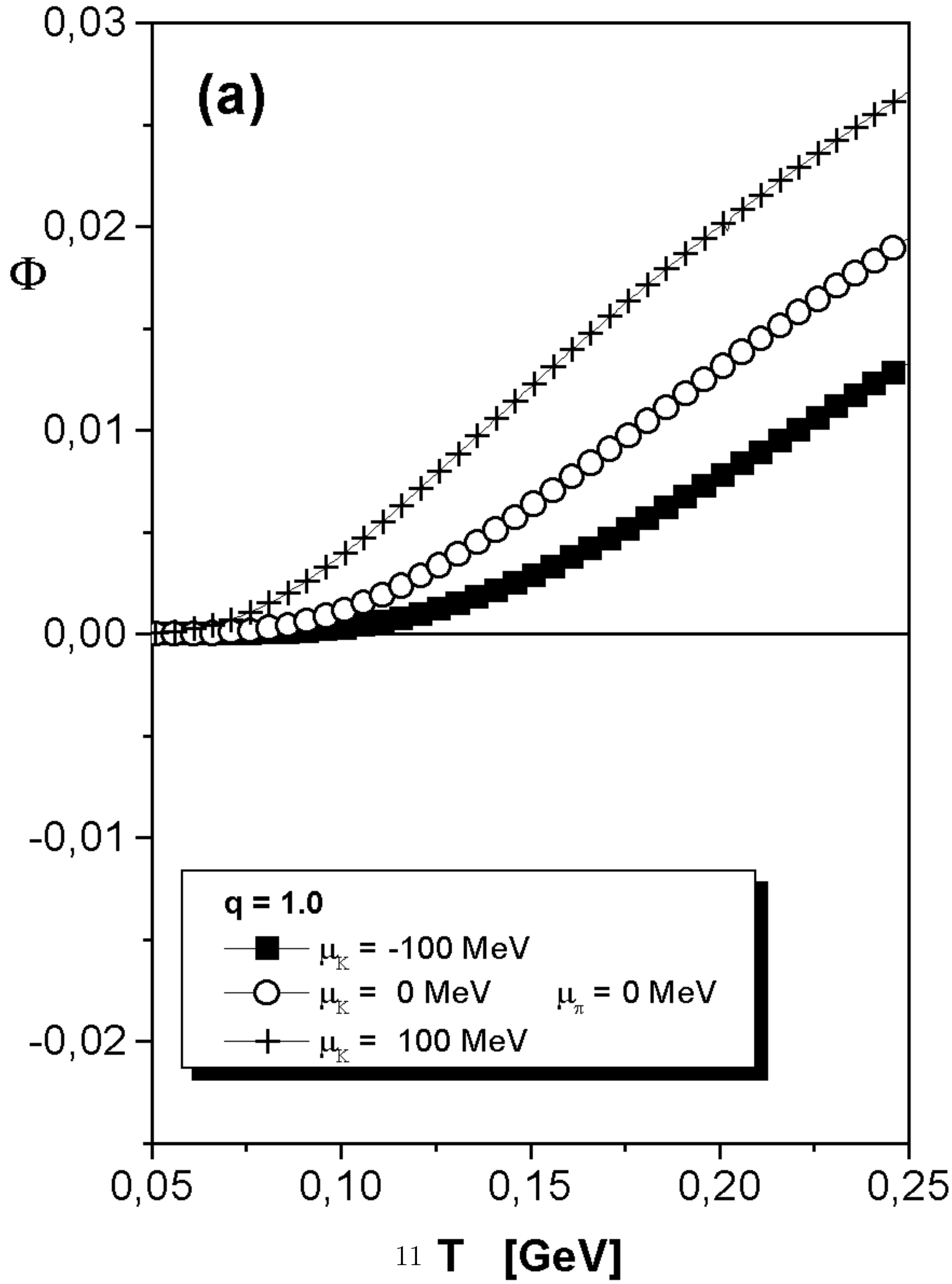
0,25
T [GeV]



(a)
0,03
0,02
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0,00
-0,01
-0,02
Φ
q = 1.0
μK = -100 MeV
μK = 0 MeV
μπ = 0 MeV
μK = 100 MeV
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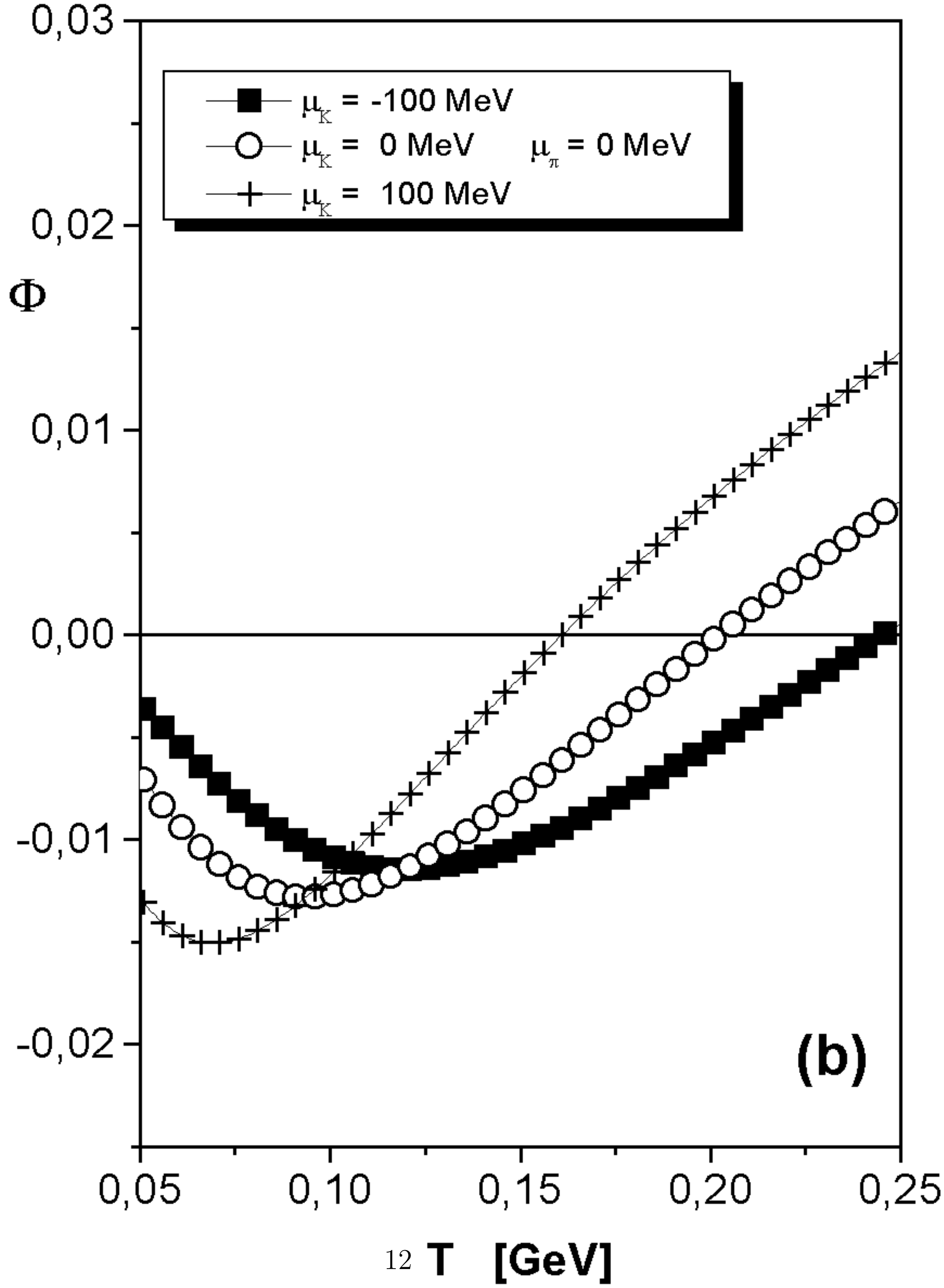
T [GeV]


μ_K = -100 MeV
μ_K = 0 MeV μ_π = 0 MeV
μ_K = 100 MeV
Φ
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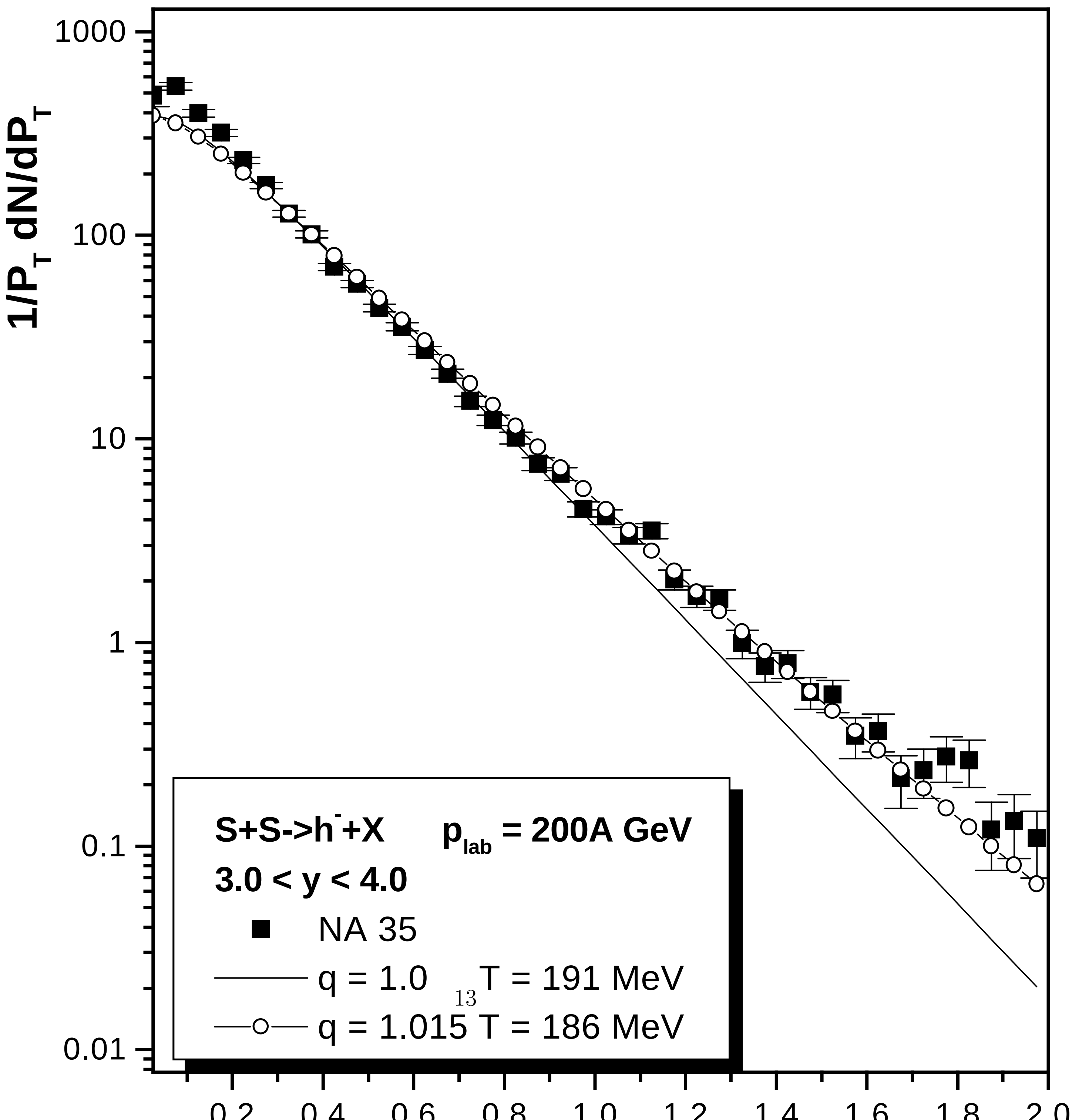
(b)
T [GeV]



1000
100
10
1
0.1
0.01
$1/P_T\ dN/dP_T$
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
S+S->h⁻+X p_{lab} = 200A GeV
3.0 < y < 4.0
NA 35
q = 1.0 T = 191 MeV
q = 1.015 T = 186 MeV